AIAA 81-0548R

The Controlled Growth Method—A Tool for Structural Optimization

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Abstract

An adaptive design variable linking scheme in a nonlinear programming (NLP) based optimization algorithm is proposed and evaluated for feasibility of application. The present scheme, based on an effectiveness measure for each variable, differs from existing methodology in that a single dominant variable controls the growth of all others in a prescribed optimization cycle. Substantial reduction in computational time, even more so for structures under multiple load conditions, coupled with a minimal accompanying loss in accuracy, vindicates the algorithm.

Contents

Development of finite element techniques for structural analysis have provided an increased capability for optimal sizing of complex structural systems. An extremely flexible design approach is available in the use of nonlinear programming methods in conjunction with finite element procedures. For a large number of design variables, constraints, and in the presence of multiple loading conditions, these techniques can be computationally costly. Approximation concepts such as design variable linking and constraint deletion have been suggested[1] to reduce the cost.

The present paper describes a "controlled growth method" that can be classified in the category of design variable linking techniques. It replaces the standard approach of working with all the design variables in a NLP methodology by a sequence of smaller optimization cycles involving a single "dominant" variable. It is also distinct from the idea of grouping variables in sets reported in literature. The empirical linking scheme is based on a defined effectiveness criterion for each design variable. User specified constraints on design variable and objective function change in a cycle provide the control to ensure convergence to an acceptable optimum.

Problem Formulation

Minimize $W(d_i)$, subject to the constraints

$$g_i(d) \leq 0 \tag{1}$$

where

$$g_i(d) = \begin{cases} w_i/w_{\text{all}} - 1 & i=1,2,\ldots j \\ \sigma_i/\sigma_{\text{all}} - 1 & i=j+1,\ldots p \\ \omega_i/\omega_{\text{all}} - 1 & i=p+1,\ldots k \end{cases} \tag{2}$$

are the j, $p-j$, and $k-p-j$ displacement, stress, and frequency constraints, respectively.

An integral feature of the proposed scheme is inherent in the way the constraints are reformulated. The k constraints are represented by a cumulative measure of constraint violation Ω as

$$\Omega = -\psi + \sum_{i=1}^{k} (\langle g_i \rangle)^r \tag{3}$$

$$\langle g_i \rangle = \begin{cases} g_i & \text{if} \quad g_i > 0 \\ 0 & \text{if} \quad g_i \leq 0 \end{cases} \tag{4}$$

r is typically 2 but is reduced sequentially to unity as the constrained optimum is approached. ψ is a small initialization value of order 10^{-2}.

A measure of effectiveness based on an optimality criteria definition[2] is adopted. The modified problem of minimum weight design subject to a single inequality constraint yields the following condition for optimality.

$$\frac{1}{\lambda_i} = \frac{\partial\Omega/\partial d_i}{\partial W/\partial d_i}\bigg|_{d_i = d_i^{\text{opt}}} = \text{const} \tag{5}$$

We define a combined measure of effectiveness (CME_i) for the ith design variable as

$$\text{CME}_i = \frac{\partial\Omega/\partial d_i}{\partial W/\partial d_i} \tag{6}$$

The variables are ordered by the magnitude of CME and only the first few above an arbitrarily prescribed cut-off value are active in the cycle. The first on this ordered list of active variables is designated as the dominant variable. The m active variables in a cycle are sized by the following linking scheme.

$$d_i^{\text{new}} = d_i^{\text{old}} + \frac{\text{CME}_i}{\text{CME}_D} \frac{d_i^{\text{old}}}{d_D} \Delta d_D \quad i=1,2,\ldots m \tag{7}$$

Δd_D is the change in the dominant variable dictated by the optimizer. The ratio CME_i to CME_D provides a proportional increment for each of the m active variables as related to Δd_D. The ratio d_i/d_D acts as a damping constant, attenuating the proportional increment in relation to the original design

Presented as Paper 81-0548 at the AIAA/ASME/ASCE/AHS 22nd Structures, Structural Dynamics and Materials Conference, Atlanta, Ga., April 6-8, 1981, submitted April 21, 1981; synoptic received Dec. 7, 1981; revision received Jan. 21, 1982. Full paper available from AIAA Library, 555 West 57th Street, New York, N.Y. 10019. Price: Microfiche, \$4.00, hard copy, \$8.00. **Remittance must accompany order**.

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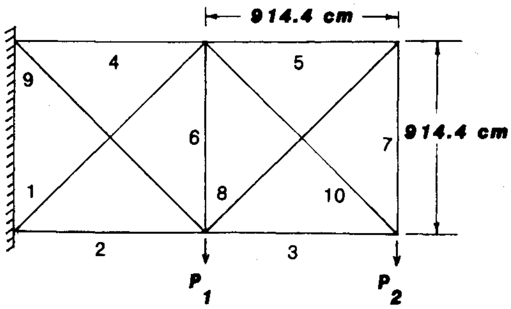

Fig. 1 Ten bar planar truss ($P_1 = P_2 = 45{,}454$ kg, $\sigma_{all} = 1761$ kg/cm^2).

Table 1 Errors in design variable and objective function values (stress constraints only)

Design variable ID number	"Exact" results, cm^2	Adaptive method results, cm^2	Errors, %
1	35.93	33.04	7.9
2	52.01	53.06	−2.0
3	25.41	24.05	5.4
4	51.21	41.15	19.6
5	0.645	2.98	—
6	0.645	0.645	0.0
7	0.645	3.06	—
8	0.645	5.70	—
9	37.06	39.42	−6.4
10	35.93	32.89	8.3
Obj. function	724.01 kg	729.00 kg	0.7

variable size. It also permits the linking of variables of one type to optimizer prescribed increments in a different variable type (i.e., membrane element thickness linked to increments in cross-sectional areas of bar elements).

A step-by-step procedure of implementing the proposed method is as follows.

1) Initialize the structure to minimum element gage.

2) For the given load conditions, compute the constraints g_i, $i = 1,2,\ldots k$. compute the cumulative constraint defined in Eq. (3).

3) Compute CME_i for all the design variables as in Eq. (6).

4) Order the design variables by decreasing magnitude of CME_i and retain the first m variables as active variables, using m which renders S minimum subject to

$$S = \sum_{i=1}^{m} \text{ABS}(\text{CME}_i) \geq f \sum_{i=1}^{n} \text{ABS}(\text{CME}_i) \tag{8}$$

where f is a judgementally chosen factor (typically f is 0.9-0.95).

5) Prescribe a constraint reduction schedule. For a structure designed subject to maximum permissible stress constraints, a logical constraint reduction schedule would correspond to a reduction in Ω equal to the constraint contribution from the m active variables in that cycle.

6) Initiate the optimization cycle with the dominant variable d_D and a prescribed constraint. The cycle is terminated with the satisfaction of the constraint or by limits on the change in the objective function value. Repeat from step 2 with an updated set of design variables.

The ten-bar truss (Fig. 1) was sized for minimum weight subject to prescribed maximum stresses in the bar elements. A comparison of results with those obtained from a conventional NLP approach are shown in Table 1. The effectiveness measure CME_i, provided a logical basis to aid in the selection of the active variables for a cycle and providing the necessary scaling factor in the proposed linking formula. At the optimum, all CME's tend to low values reflecting little potential for change in the design. Similar results for structures with stress, displacement, and frequency constraints have been obtained.[3]

The proposed controlled growth method permits significant savings in the computational effort stemming largely from a reduced number of gradient evaluations. These savings can be dramatic for structures with a large number of design variables and under multiple loading conditions.[3] The effectiveness of the cumulative constraint formulation is demonstrated. The scheme has been shown to be equally effective[3] for structures comprising of mixed element types. A decreased core requirement for the optimization program adds to the efficiency of the method.

References

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